Lines and Exponential Functions

Finite Math

6 February 2017

Definition

Definition (Line)

A line is the graph of an equation of the form

$$Ax + By = C$$

where not both of A and B are equal to zero (i.e., if $A = 0$, then $B \neq 0$ and vice-versa).

Graphing Lines

There are two common ways of graphing lines: by **finding intercepts** and by **using the slope and a point.** We will focus on the method of finding intercepts here in the notes. You can read about using the slope to graph a line in the textbook.

Finding Intercepts

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Every line will have at least one intercept, but most have two. There are three special cases in which the line has only one intercept: if $A = 0$, $B = 0$, or $C = 0$. We will return to these special cases in a little bit.

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Graphing with Intercepts

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Example

Graph the line $4x - 3y = 12$ *using intercepts.*

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Graphing a Line Through the Origin

Example

Graph the line $2x + 3y = 0$.

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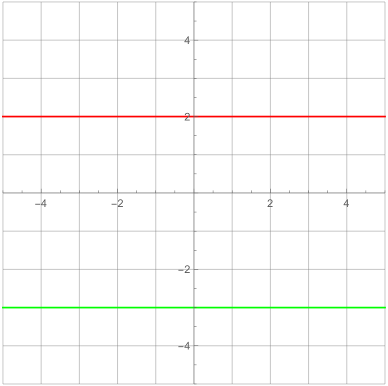
If $A = 0$, we end up with the line $y = \frac{C}{B}$, which is a horizontal line where every y-value is $\frac{C}{B}$. A special one of these is when C is also zero so we get the equation $y = 0$. The graph of this line is the x-axis.

Horizontal and Vertical Lines

Here are the graphs of $y = 2$ (red) and $y = -3$ (green).



Horizontal and Vertical Lines

The cases when $A = 0$ or $B = 0$ in $Ax + By = C$ correspond to horizontal and vertical lines, respectively. If $B = 0$, we end up with the line $x = \frac{C}{A}$, which is a vertical line where every x-value is $\frac{C}{A}$. A special one of these is when C is also zero so we get the equation $x = 0$. The graph of this line is the y-axis.

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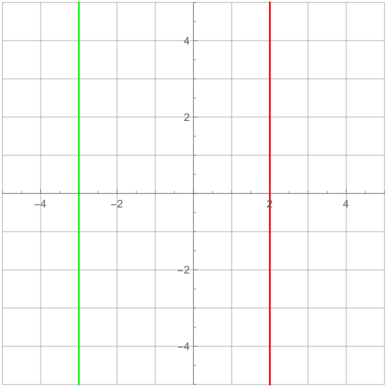

Now You Try It!

Example

Graph the following lines:

(a)

$$2x - y = 3$$

(b)

$$2x + 4y = 8$$

(c)

$$3x - 2y = 0$$

(d)

$$6x = 18$$

Definition

Definition (Exponential Function)

An exponential function is a function of the form

$$f(x) = b^x,\ b > 0,\ b \neq 1.$$

b is called the base.

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  an imaginary number! This kind of thing will always happen if b is negative.
- If $b = 0$, then for negative x values, f is not defined. For example,

$$f(-1) = 0^{-1} = \frac{1}{0} = \text{undefined.}$$

Graphing Exponential Functions

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Sketch the graph of $f(x) = 2^x$.

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Example

Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$.

Negative Powers

Notice that

$$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$$

so that when $b < 1$, we can set $b = \frac{1}{c}$ and have $c > 1$ and

$$f(x) = b^x = \left(\frac{1}{c}\right)^x = c^{-x}.$$

So, we can always keep the base larger than 1 by using a minus sign in the exponent if necessary.

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Property (Graphical Properties of Exponential Functions)

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5. b^x *is decreasing if* $0 < b < 1$.

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2. $a^x = a^y$ *if and only if* $x = y$
3. $a^x = b^x$ *for all* x *if and only if* $a = b$